Theory notes related to the concept of polywell

Lorenzo Fortunato - Univ. of Padova (Italy)

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Magnetic field generated by a current loop

The magnetic field generated at a generic point in space P, by a current loop (a circular coil) of intensity i and radius R is found by integrating the Ampère-Laplace law:

$$d\vec{B} = \frac{\mu_0 i}{4\pi}\frac{d\vec{s}\times\vec{u}_r}{r^2} = \frac{\mu_0 i}{4\pi}\frac{\vec{ds}\times\vec{r}}{r^3} \tag{1}$$

where μ_0 is the vacuum magnetic permeability, $\vec{ds}$ is an element of length the current loop located at the point C on the circumference with coordinates $(R\cos(\theta), R\sin(\theta), 0)$, $\vec{r}$ is the vector distance of the element from a generic point in space P with coordinates (x_0, y_0, z_0) with respect to the origin placed in the center of the loop. The xy plane contains the loop and the z axis is perpendicular to it.

We can write

$$\vec{r} = (x_0 - R\cos(\theta), y_0 - R\sin(\theta), z_0) = \sqrt{(x_0 - R\cos(\theta))^2 + (y_0 - R\sin(\theta))^2 + z_0^2} \tag{2}$$

and, assuming an anticlockwise direction along the circumference,

$$d\vec{s} = (-R\sin(\theta)d\theta, R\cos(\theta)d\theta, 0) \tag{3}$$

one can calculate the vector product

$$d\vec{s}\times\vec{r} = Rd\theta \begin{vmatrix} \vec{u}_x & \vec{u}_y & \vec{u}_z \\ -\sin(\theta) & \cos(\theta) & 0 \\ x_0 - R\cos(\theta) & y_0 - R\sin(\theta) & z_0 \end{vmatrix} \tag{4}$$

The magnetic field at the point P is thus

$$\vec{B} = \frac{\mu_0 i}{4\pi} R \int_0^{2\pi} d\theta \frac{\vec{u}_x\cos(\theta)z_0 + \vec{u}_y\sin(\theta)z_0 + \vec{u}_z(R - y_0\sin(\theta) - x_0\cos(\theta))}{((x_0 - R\cos(\theta))^2 + (y_0 - R\sin(\theta))^2 + z_0^2)^{3/2}} \tag{5}$$

This integral can be performed numerically, except for points sitting on the coil itself where it diverges. Notice that, on the plane containing the coil, $\vec{B}$ points along the z-axis and it is directed up for points inside the circle and down for points outside it. Its value at the center of the loop is:

$$\vec{B}(P = O) = \frac{\mu_0 i}{4\pi} R \int_0^{2\pi} d\theta \frac{R}{R^3} \vec{u}_z = \frac{\mu_0 i}{2R} \vec{u}_z \tag{6}$$

as expected. For a point on the z-axis we have instead:

$$\vec{B}(P = (0, 0, z_0)) = \frac{\mu_0 i}{2} \frac{R^2}{(R^2 + z_0^2)^{3/2}} \vec{u}_z \tag{7}$$

as expected too.

The present derivation is made under the assumption that the loop is a circumference of negligible thickness (i.e. the wire is extremely thin). Of course the formula found above does not give a perfectly correct result for magnetic fields in close proximity of the wire. In that case, one should introduce the details of shape and lengths of the conducting wires. Formula (5) works well for distances from the wires that are bigger than the wire's radius and it can, therefore, be considered an excellent approximation for all practical considerations on the polywell.